RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous Degree College with P.G. Section under University of Calcutta)

B.A./B.SC. SECOND SEMESTER EXAMINATION, MAY 2011

FIRST YEAR

Date : 31/05/2011 **MATHEMATICS (General)**

Time : 11 am – 2 pm Paper : II Full Marks : 75

[Use Separate Answer Scripts for each group]

Group – A

Answer Q.No. 1 and any one from the Q.No. 2 and Q.No. 3 :

1. a) Find the polar coordinates of the mid point of PQ when $P \equiv \left(5, \frac{\pi}{4}\right),\ Q \equiv \left(7, -\frac{3\pi}{4}\right)$

Or,

Find the nature of the conic $r = \dfrac{21}{5 - 2\cos\theta}$ [1]

b) Find the fixed point of the rigid motion

$$x' = \frac{4}{5}x - \frac{3}{5}y + 2$$

$$y' = \frac{3}{5}x + \frac{4}{5}y - 2$$

Or,

By shifting the origin to the point (α, β) without changing the direction of the axes, each of the equations x – y + 3 = 0 and 2x – y + 1 = 0 is reduced to the form $ax' + by' = 0$, find α and β. [2]

2. a) Reduce the equation $x^2 - 2xy + y^2 + 6x - 14y + 29 = 0$ to its canonical form and state the nature of the conic represented by it. [6]

b) Show that the straight line $r\cos(\theta - \alpha) = p$ touches the conic $\frac{\ell}{r} = 1 + e\cos\theta$

if $(\ell\cos\alpha - ep)^2 + \ell^2\sin^2\alpha = p^2$ [6]

3. a) Find the equation to the pair of straight lines joining the origin to the point of intersection of the line y = mx + c and the curve $x^2 + y^2 = a^2$. [4]

b) Find the locus of the poles of the tangents to the parabola $y^2 = 4ax$ w.r.t $y^2 = 4bx$. [4]

c) Find the polar equation of the line joining the points (r_1, θ_1) and (r_2, θ_2) [4]

Group – B

Answer Q.No. 4 and any one from the Q.No. 5 and Q.No. 6 :

4. a) Find the value of $-5\hat{j} \cdot (\hat{k} \times \hat{i})$

Or,

Write down the vector equation of the straight line passing through the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ [1]

b) Find the vector perpendicular to both the vectors $2\hat{i} + \hat{j} - 3\hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$

Or,

Find the work done in moving an object along straight line from (3, 2, –1) to (2, –1, 4) in a force field given by $\vec{F} = 4\hat{i} - 3\hat{j} + 2\hat{k}$. [2]

5. a) Prove by vector method that the semi-circular angle is a right angle. [4]

b) Prove the identity $[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times a]=[\vec{a} \ \ \vec{b} \ \ \vec{c}]^2$ [4]

c) Find the moment of the force $4\hat{i}+2\hat{j}+\hat{k}$ through the point (5, 2, 4) about the point (3, –1, 3) [4]

6. a) Prove the trigonometrical formula $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$ by vector method, where a, b, c and A have their usual meanings in a triangle ABC. [5]

b) Find the equation of the plane passing through the origin and parallel to the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $4\hat{i}-5\hat{j}+4\hat{k}$. [3]

c) Find the volume of the tetrahedron ABCD, where the position vectors of A, B, C and D are (–1,1,1), (1,–1,1), (1,1,–1) and (4,1,–3) [4]

Group – C

Answer Q.No. 7 and any two from the Q.No. 8 to Q.No. 10 :

7. Anwer any one question :

a) Find all the asymptotes of $x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$ [5]

b) Find the envelope of the family of straight lines $\dfrac{x}{a}+\dfrac{y}{b}=1$ where a, b are variable parameters connected by the relation a + b = c, c being a nonzero constant. [5]

c) i) Prove that (1, 0) is a node on the curve $x^4 - 2y^3 - 3y^2 - 2x^2 + 1 = 0$

ii) Show that (0,0) is a double point on the curve $x^3 + y^3 - 3axy = 0$. Find the nature of the double point. [2+3]

8. a) Prove that a convergent sequence of real numbers has a unique limit. [3]

b) Examine whether the sequence $\left\{\dfrac{4n+3}{3n+4}\right\}$ is monotone increasing or monotone decreasing. [2]

c) State Leibnitz's theorem on alternating series. [2]

d) Examine the convergence of the infinite series : $\dfrac{1}{2}+\dfrac{2}{2^2}+\dfrac{3}{2^3}+........$ [3]

9. a) State and prove Lagrange's mean value theorem. [4]

b) Expand sin x using Maclurin's infinite series stating the range of validity of x. [4]

c) Explain whether the Rolle's theorem is applicable to the function |x| in any closed interval containing origin. [2]

10. a) Evaluate $\lim\limits_{n\to\infty}\left(1+\dfrac{3}{n}\right)^{2n}$ [3]

b) Find the points of local extremum of the function $f:R\to R$ defined by $f(x) = 12x^5 - 45x^4 + 40x^3 + 1$, $x\in R$ [3]

c) Examine the existence of maxima or minima of the function f(x, y) = xy ; $x,y\in R$ subject to the condition 5x + y = 13 [4]

Group – D

Answer any one question :

11. a) Evaluate any one :

i) $\displaystyle\int\frac{\cos x}{2\sin x+5\cos x}dx$

ii) $\int \frac{x^2-1}{x^4-x^2+1}dx$ [4]

b) Evaluate : $\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)......\left(1+\frac{n}{n}\right)\right\}^{1/n}$ [4]

c) State the Fundamental Theorem of Integral Calculus. [2]

12. a) If $I_n = \int_0^{\pi/4} \tan^n x\,dx$, show that $I_{n+1} + I_{n-1} = \frac{1}{n}$ where n is a positive integer greater than 1. [4]

b) Prove that $\int_0^{\pi/2} (a^2\cos^2 x + b^2\sin^2 x)dx = \frac{\pi}{4}(a^2+b^2)$ [3]

c) Prove by the definition of the integration $\int_0^1 x^2 dx = \frac{1}{3}$ [3]

Group – E

Answer any one question :

13. a) Find the order and degree of the differential equation : $\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right)^5 + 10y = 0$ [2]

b) Solve : $\frac{dy}{dx} = \frac{x+y+1}{3(x+y)+1}$ [4]

c) Test whether the differential equation $(2x^3 + 4y)dx + (4x + y - 1)dy = 0$, is exact or not, and hence solve it. [4]

14. a) Find the solution of the equation $(x^2 + y^2 + 2x)dx + 2y\,dy = 0$ when $x = y = 1$ [2]

b) Solve the equation $y = px + \sqrt{a^2p^2 + b^2}$, $p \equiv \frac{dy}{dx}$ and obtain the singular solution. [4]

c) Solve : $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$ [4]